COPERNICUS

A TRADITION OF EXCELLENCE

WELCOME TO COPERNICUS
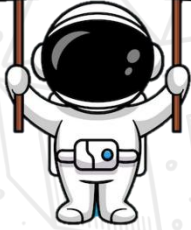
INSTRUCTIONS

You are about to take Copernicus Exam.

Please read the followings carefully.

1. The exam has 25 multiple choice-questions. Each question weighs 4 points. The maximum score a student can get is 100. There is a penalty of one point for each incorrect answer. So only answer the questions you are sure of.

2. Start with the easier questions, you can always come back to the questions you leave.

3. The time allocated for the exam is 60 minutes. You will start when the invigilator tells you to start.

4. You are required to comply with the directions given by the head invigilator before the examination.

5. Those who are taking the exam with a mobile phone MUST make sure that during the examination no one calls.

6. If anything in the examination is unclear, you can contact the invigilator.

7. Where permitted you may use a translation dictionary.

8. Students must not give or receive assistance of any kind during the exam. Any cheating, any attempt to cheat, assisting others to cheat, participating therein, or engaging in such improper conduct is a serious violation and will generally result in disqualifying.

Remember that "Hard work beats talent when talent doesn't work hard"
We wish you the very best luck on the exam.

1. $A7BC$ and $A8BC$ are four-digit natural numbers. Knowing that dividing $A7BC$ by 12 it leaves a remainder of 4, what is the remainder of dividing $A8BC$ by 12?

A) 4

B) 6

C) 8

D) 10

E) 12

2. A number AB is chosen at random from the set of two-digit numbers. What is the probability that the number $\sqrt{2.\overline{AB}}$ is rational?

A) $\dfrac{1}{10}$

B) $\dfrac{1}{15}$

C) $\dfrac{1}{30}$

D) $\dfrac{2}{45}$

E) $\dfrac{3}{50}$

3. Let's define $\Delta(x)$ as
$\Delta(x) = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \cdots + \sqrt{x}$.
For example:
$\Delta(6) = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6}$.
According to this, what is the result of the expression below?

$$\frac{\Delta(64) - \Delta(63)}{\Delta(16) - \Delta(15)}$$

A) 1

B) 2

C) 3

D) 4

E) 5

4. What is the sum of the integers x that satisfy the inequality below?

$$\frac{1}{|x + 4| - 2} > \frac{1}{4}$$

A) – 44

B) – 39

C) – 33

D) – 27

E) – 24

5. Knowing that $-3 \leq x \leq 4$. What is the sum of the smallest and largest values that the expression $x^2 - 4x$ can assume?
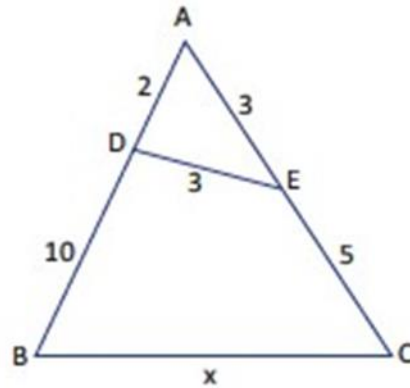
A) 16

B) 17

C) 18

D) 19

E) 20

6. The graph shows the variation of the heights of plants A and B as a function of the time in months. According to this, what will be the difference in height between the two plants in the seventh month?
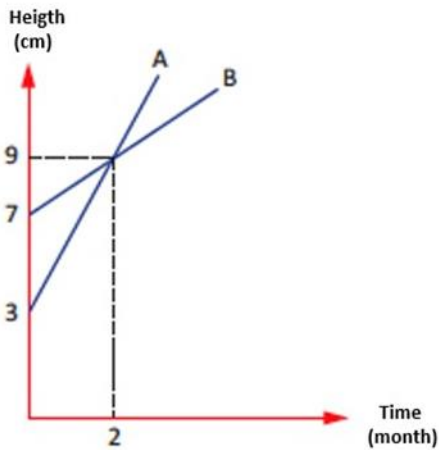
A) 9

B) $\dfrac{19}{2}$

C) 10

D) $\dfrac{21}{2}$

E) 11

7. Given the triangle ABC and its measures shown in the figure, what is the value of $|BC| = x$?

A) 8

B) 10

C) 12

D) 14

E) 16

8. Considering the inequalities below, which numerical range represents all possible values for the expression $x^2 + y^3$?
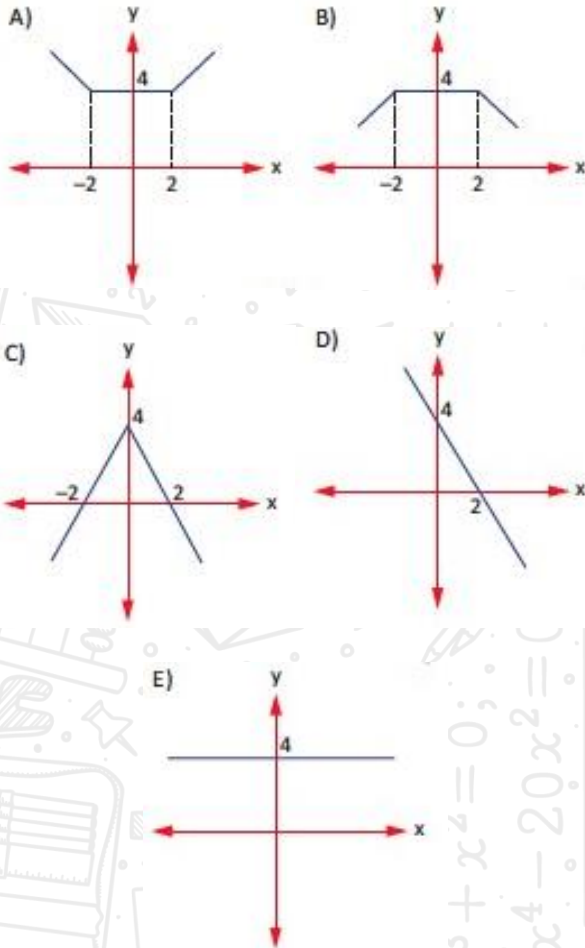
$$-2 \le x \le 3$$
$$-1 \le y \le 2$$

A) $[-1, 17]$

B) $[3, 17]$

C) $[-1, 13]$

D) $[3, 13]$

E) $[-3, 17]$

9. Which of the following is the graph of the function below?

$$f: \mathbb{R} \to \mathbb{R}, f(x) = |x - 2| + |x + 2|$$
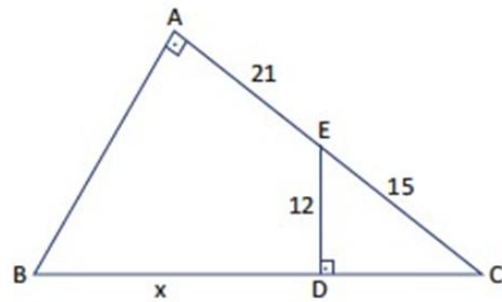
A)

B)

C)

D)

E)

10. Since $\sqrt{5} < x\sqrt{6}$, which of the following could be the value of x?

A) $\dfrac{5}{6}$

B) $\dfrac{8}{9}$

C) $\dfrac{9}{10}$

D) $\dfrac{10}{11}$

E) $\dfrac{11}{12}$

11. If the sum of six integers is divisible by 6, at most how many of these numbers are not divisible by 6?

A) 6

B) 5

C) 4

D) 3

E) 2

12. BAC and EDC are right triangles in A and D, respectively, such that $|AE| = 21\ cm$, $|EC| = 15\ cm$, and $|ED| = 12\ cm$. If x is the length of BD in centimeters, what is the value of x?

A) 42

B) 47

C) 51

D) 56

E) 60

13. An ant, that was at point A of the rectangular prism shown in the image below, walked $x\ cm$ to point B. Given that the ant arrived at point B by the shortest possible path across the surface of the prism, what is the value of x?

8 cm
3 cm
3 cm
B
A
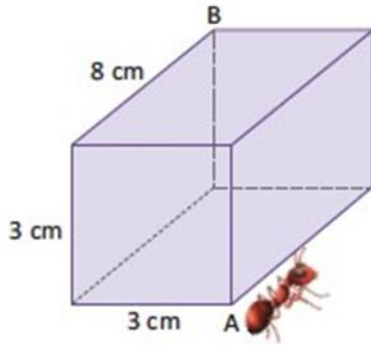
- **A)** 8
- **B)** 9
- **C)** 10
- **D)** $\sqrt{120}$
- **E)** $\sqrt{130}$

14. A square cardboard with a side length of $36\ cm$ is given in the figure. This entire cardboard is cut into 11 squares whose side lengths are integers. What is the perimeter of the largest square that can be obtained from the 11 squares obtained?

36 cm

- **A)** 18
- **B)** 24
- **C)** 36
- **D)** 64
- **E)** 72

15. In a school with 790 students, a total of 32 classes will be formed. Each class must have 12, 18 or 28 students, requiring at least one class of each of these three sizes. Knowing this, how many classes of 28 students will be formed?
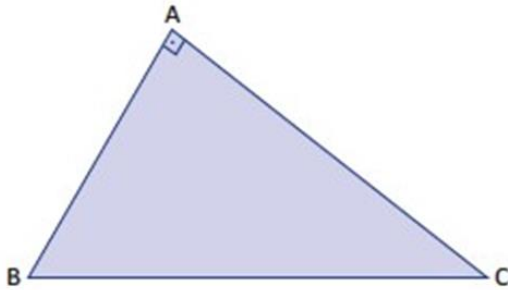
- **A)** 24
- **B)** 25
- **C)** 26
- **D)** 27
- **E)** 28

16. Let x, y, and z be real numbers, with $x + y + z = 9$. From this, what is the smallest value that the expression below can take?

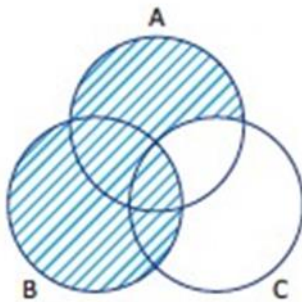$$\sqrt{x^2 + 4} + \sqrt{y^2 + 16} + \sqrt{z^2 + 36}$$

- **A)** 10
- **B)** 12
- **C)** 14
- **D)** 15
- **E)** 18

17. How many different right triangles are there such that all sides are integer lengths and at least one of them is 15 cm long?

A) 1

B) 2

C) 3

D) 4

E) 5

18. Which of the following represents the shaded region in the figure?

A) $B \cup (C - A)$

B) $B \cup (A - C)$

C) $(A \cap B) - C$

D) $B \cup (A \cap C)$

E) $(A \cup B) - (A \cap C)$

19. A number is said to be a perfect number if it is equal to the sum of all its positive divisors except itself. For example, 6 is a perfect number because the positive divisors of 6 are 1, 2, 3, and 6, and if we add all the divisors except itself, we get $1 + 2 + 3 = 6$. Based on this definition, which of the following is a perfect number?

A) 108

B) 72

C) 42

D) 36

E) 28

20. If $a^3 = 400 \cdot b$, with a and b being positive integers, what is the smallest possible value of the product of $a \cdot b$?

A) 100

B) 200

C) 400

D) 800

E) 1600

21. If $\dfrac{2x+y}{x-y} = 3x$, where x and y are positive integers with $12 < x < 20$, what is the value of the product $x \cdot y$?

A) 36

B) 48

C) 64

D) 72

E) 1084

22. How many whole numbers multiples of 3 are there between 1 and 2005?

A) 664

B) 665

C) 667

D) 668

E) 669

23. The graph shows the distance traveled by the two vehicles as a function of time. Assuming that the two vehicles started from the same place and at the same time, after how many hours will the distance between them be exactly 40 km?
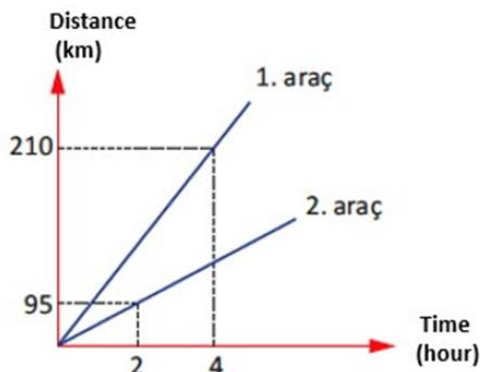
A) 8

B) 9

C) 10

D) 12

E) 14

24. To enclose a rectangular plot of 60 meters squares with a fence formed by two strands of wire, 64 meters of wire were used. What is the difference between the length and width of the field?

A) 4 meters

B) 7 meters

C) 11 meters

D) 17 meters

E) 28 meters

25. Ann, Paul and Mary try to guess how many balls are inside a closed box. They already know that this number is greater than 100 and lower than 140. They do the following statements:
- Ann: In the box there are more than 100 balls and less than 120 balls.
- Paul: There are more than 105 balls and less than 130 balls in the box.
- Mary: In the box there are more than 120 balls and less than 140 balls.

If only one of these statements is known to be correct, how many possible values are there for the number of balls inside the box?

A) 1

B) 5

C) 11

D) 13

E) 16