A TRADITION OF EXCELLENCE

INSTRUCTIONS

You are about to take Copernicus Exam.

Please read the followings carefully.

1. The exam has 25 multiple choice-questions. Each question weighs 4 points. The maximum score a student can get is 100. There is a penalty of one point for each incorrect answer. So only answer the questions you are sure of.
2. Start with the easier questions, you can always come back to the questions you leave.
3. The time allocated for the exam is 60 minutes. You will start when the invigilator tells you to start.
4. You are required to comply with the directions given by the head invigilator before the examination.
5. Those who are taking the exam with a mobile phone MUST make sure that during the examination no one calls.
6. If anything in the examination is unclear, you can contact the invigilator.
7. Where permitted you may use a translation dictionary.
8. Students must not give or receive assistance of any kind during the exam. Any cheating, any attempt to cheat, assisting others to cheat, participating therein, or engaging in such improper conduct is a serious violation and will generally result in disqualifying.

Remember that “Hard work beats talent when talent doesn't work hard”

We wish you the very best luck on the exam.

1. It is known that $P(x)$ leaves remainder $(x+3)$ when divided by (x^2-3x+2) and remainder $(3x-5)$ when divided by (x^2-x-2). So, which of the following is the remainder when dividing the polynomial $P(x)$ by (x^2-1)?

A) $3x+1$
B) $6x-2$
C) $6x+2$
D) $3x-5$
E) $5x-1$

2. Knowing that $P(x)=x^2-2x-24$, which of the following is the quotient obtained when the sum $P(x+5)+P(x-5)$ is divided by $x-1$?

A) $2x-2$
B) $5x-10$
C) $3x-6$
D) $x-1$
E) $2x-4$

3. Considering that the zeros of the polynomial below are -3, 5, a, and b, with $-3<a<5$, what is the remainder of the division of the polynomial $P(3x-2)$ by $(x-1)$?

$$P(x)=x^4+px^3+rx^2+nx+90$$

A) 48
B) 56
C) 72
D) 84
E) 96

4. Given that the equation below has two distinct real roots, what is the sum of the positive integer values that m can take?

$$(m-2)x^2-2mx+m+6=0$$

A) 1
B) 2
C) 3
D) 6
E) 10

5. The equation $x^2-6x+m=0$, where m is an integer, has two distinct non real roots. According to this, how many distinct quadratic equations are there with the form $x^2+8x-6+m=0$ and with two distinct real roots each?

A) 8
B) 9
C) 10
D) 11
E) 12

6. The golden ratio is the harmony observed between the parts of a whole in science, art and nature. It is a geometric and numerical relationship that can be expressed by the equation $x^2-x-1=0$, with m, the largest root of the equation, being the golden ratio. Being p and r positive integers, we have $m^4=pm+r$. Knowing this, what is the value of the sum $p+r$?

A) 5
B) 4
C) 3
D) 6
E) 10

7. Which of the following is an equation whose roots are the reciprocals of the roots of the equation below?

$$3x^2 - 4x - 2 = 0$$

A) $2x^2 - 4x - 3 = 0$
B) $x^2 - 4x - 3 = 0$
C) $2x^2 + 4x - 3 = 0$
D) $2x^2 - 3x + 4 = 0$
E) $3x^2 + 2x + 4 = 0$

8. The roots of the equation below are x_1 and x_2. If $(x_1 + 4) \cdot (x_2 + 4) = 39$, then what is the value of a?

$$x^2 - ax + 7 = 0$$

A) 3
B) 4
C) 5
D) -3
E) -4

9. The roots of the equation below are x_1 and x_2. Which of the following could be a quadratic equation with roots $3x_1 - 2$ and $3x_2 - 2$?

$$x^2 - 4x - 7 = 0$$

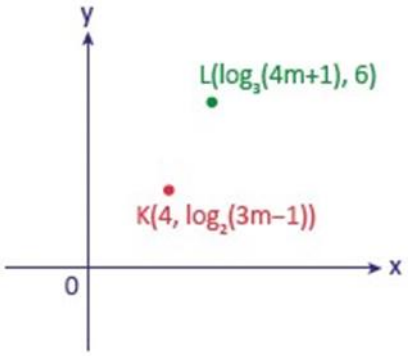
A) $x^2 - 8x + 23 = 0$
B) $x^2 + 8x - 47 = 0$
C) $x^2 - 17x + 83 = 0$
D) $x^2 - 8x - 83 = 0$
E) $x^2 - 3x + 14 = 0$

10. How many integers are in the largest domain of the function below?

$$f(x) = \frac{\log(x-5)}{\ln(18-x)}$$

A) 11
B) 10
C) 9
D) 8
E) 7

11. The positions of K and L points on the cartesian plane are shown in the image. Accordingly, what is the value of the integer m?



A) 19
B) 20
C) 21
D) 22
E) 24

12. If $\log_2(4x - 3) - \log_2(2x + 3) = 0$, then what is the value of $\log_2(\log_9 x)$?

A) -1
B) -2
C) 2
D) 3
E) 4

13. Consider that $log_a(b) = \frac{log_c(b)}{log_c(a)}$, for $a, b, c \in \mathbb{R}^+$ and $a \neq 1$, $c \neq 1$. If $m = \log_2 11$ and $n = \log_{11} 3$, which of the following is the equivalent of $\log_{33} 6$ in terms of m and n?

A) $\frac{1+mn}{m+mn}$

B) $\frac{m+mn}{n+nm}$

C) $\frac{1+mn}{n+nm}$

D) $\frac{m+n}{mn+1}$

E) $\frac{1+mn}{m-n}$

14. On the Richter scale, the magnitude R of an earthquake is calculated using the expression $R = \log T$, where T is the destructive power of the earthquake. So how many times more destructive is a magnitude 7.8 earthquake compared to a magnitude 5.8 earthquake, both measured on the Richter scale?

A) 1

B) 10

C) 50

D) 75

E) 100

15. In a factory that produces plastic goods, 30% of the daily production is made by A machines, 20% by B and 50% by C machines. The percentage of defective output of these machines are 3%, 4%, and 2%, respectively. Given that a randomly selected plastic item at the end of a production day is defective, what is the probability that it was produced by machine B?

A) $\frac{8}{27}$

B) $\frac{5}{26}$

C) $\frac{9}{23}$

D) $\frac{4}{18}$

E) $\frac{3}{14}$

16. Paul draws a card from a bag containing 9 cards numbered 1 to 9. If the drawn card is not an odd number, he holds it in his hand and draws a new card. This process continues until an odd number appears for the first time. According to this, what is the probability that the odd number is drawn on Paul's third try?

A) $\frac{5}{42}$

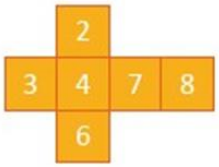
B) $\frac{10}{13}$

C) $\frac{11}{42}$

D) $\frac{5}{9}$

E) $\frac{4}{21}$

17. The numbers shown in the figure are written on the faces of the cube and the regular tetrahedron, whose open forms are given in the images. These shapes are folded to form a regular tetrahedron and cube, and then thrown into the air. What is the probability that the sum of the numbers on the visible faces of the two shapes when they fall on a flat surface is 37?



A) $\frac{1}{24}$

B) $\frac{1}{16}$

C) $\frac{1}{12}$

D) $\frac{1}{8}$

E) $\frac{1}{6}$

18. For the widest domain of the function, how many integer values are there in the range set of $f(x)$?

$$f(x) = |x - 3| - |x + 7|$$

A) 18

B) 19

C) 20

D) 21

E) 22

19. The function $f(x)$ is periodic and has a period equal to 7. It is also known that $f(8) = 9$ and $f(2) = 13$. So, what is the value of the difference $f(16) - f(1)$?

A) 22

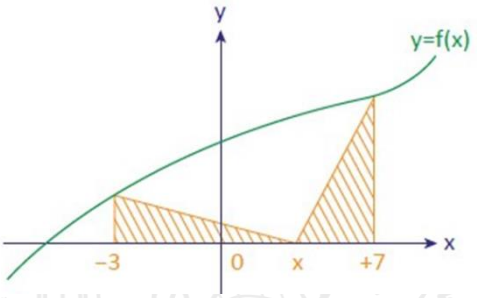
B) 13

C) 9

D) 4

E) -4

20. Let $g(x)$ be the sum of areas of the shaded regions in the graph, from -3 to x and from x to 7. A function $f(x)$ is defined. Knowing that $g(2) = 20$, what is the value of the sum $f(-3) + f(7)$?



A) 12

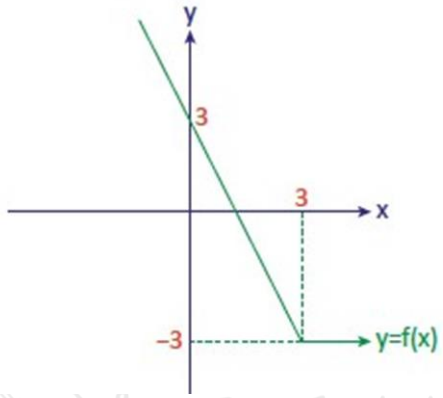
B) 10

C) 8

D) 7

E) 6

21. Which of the following is the function $y = f(x)$ graphed in the figure?



A) $|x-3|+x$

B) $|x+3|-x$

C) $|x-3|$

D) $|x-3|-|x|$

E) $|x-3|-x$

22. Diane has 2 candies less than Heidi and 5 candies more than Tom. Which of the following must be true for all three friends to have the same amount of candy?

A) Heidi should give 3 candies and Diane 2 candies to Tom.

B) Heidi should give 4 candies to Tom.

C) Heidi should give 2 candies and Diane 1 candy for Tom.

D) Heidi should give 3 candies and Diane should give 1 candy to Tom.

E) Tom should give one candy to Diane and get 4 candies from Heidi.

23. If a and b are integers such that $2a = b^3$ and $a^{b-1} = 4$, what is the value of the difference $2a - 3b$?

A) -3

B) -1

C) 0

D) 1

E) 2

24. The pointer on a scale shows the weight, in kilograms, of objects placed on it. Figure 1 shows the scale pointer after an empty plate has been placed on it. Figure 2 shows the scale pointer after 4 equal pieces of cheese have been placed on the plate. So, what is the weight in kilograms of one of these pieces of cheese?

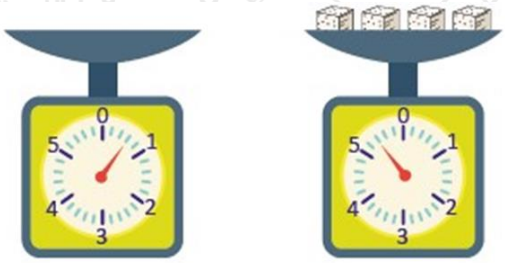


Figure 1 Figure 2

A) 1

B) 1.2

C) 1.4

D) 1.5

E) 1.6

25. Orange juice produced in a fruit juice factory is stored in 1.5 liter glass bottles or 2.5 liter plastic gallon. For this factory, the total cost of a glass bottle of orange juice is \$3.7 and the total cost of a plastic gallon of orange juice is \$4.5. Also, the cost of the glass bottle to this factory is \$0.2 more than the cost of the plastic gallon. So what is the cost of 1 liter of orange juice for this factory?

A) 0.9
B) 1
C) 1.2
D) 1.3
E) 1.4